Construction Zero Cross Correlation Code using Permutation Matrix for SAC-OCDMA Systems

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Abstract—**This paper present a new method for constructing zero cross correlation code with the help of permutation matrices. The benefits of this newly proposed code are easy way code construction, the code weight exist for every natural number and the code length is acceptable. The numerical comparison shows that the proposed code has better or compatible code length compared with other existing zero cross correlation code in Optical Spectrum Code Division Multiple Access (OSCDMA) systems.**

Keywords— Spectral Amplitude coding (SAC), Optical Code Division Multiple Access (OCDMA), Zero cross-correlation (ZCC) code, Permutation Matrix.

I. Introduction

The main feature that distinguishes optical code division multiple access (OCDMA) from other multiple access techniques is the use of so-called orthogonal codes to allow multiple users to utilize the same overlapping spectral range without interfering with each other. An OCDMA system interrupts from different noises like as shot noise, thermal noise, dark current and multiple access interference (MAI) from other users. Surrounded by all these noises, MAI is considered as the one of the dominated source in the system. Hence, the design of code sequence is important to reduce the impact of MAI to the total received power [1]. However the multiple access interference can be cancelled by balance detection scheme, a phase induced intensity noise (PIIN) rising from spontaneous emission of broadband source, integrally remains. So the studies bring an care to develop the code system in which the effect of multiple access interference and phase induced intensity noise of the total received power is reduced [2, 3, 4].

In Optical Spectrum Code Division Multiple Access (OSCDMA) systems, several codes proposed in literature such as optical orthogonal codes (OOC) [5], modified double weight (MDW) [6] and modified frequency-hopping (MFH) [7] codes. Nevertheless, these codes suffer from many limitations one way or another. The codes' constructions are either complicated (e.g., OOC and MFH codes), the cross-correlation are not ideal or the code length is too long (e.g., OOC and Prime code). Long code length is another disadvantage since either very wide band sources or very narrow filter bandwidths are required. Therefore, the researcher start thinking about to design the Zero Cross-Correlation (ZCC) codes to fulfil this property [6, 8, 9, 10, 12]. The advantages of codes with zero cross correlation have less noise, which results in reducing the hardware complexity. In this paper, we present a new zero cross correlation codes with the help of permutation matrices called Permutation Matrix Zero Cross-Correlation (PM-ZCC) codes. The benefits of the proposed new PM-ZCC codes are: i) any positive integer number of code weight; ii) cross correlation is equal to zero. The paper is organized as follows: The simple concept of permutation matrix, double weight code, modified double weight code; their properties and process of constructing codes are given in section II. The code development and properties of the proposed code presented in Section III. The comparison of proposed code with other existing zero cross correlation codes and discussions are given in Section IV. The conclusion drawn in in Section V.

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I. Definitions, notations and results

A. Permutation Matrix

For the A permutation matrix is a square matrix achieved from the same size identity matrix by a permutation of rows [11]. Such a matrix is always row equivalent to an identity. The examples of 2x2 permutation matrices are:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

which means each row and column of a permutation matrix contain exactly one non-zero entry, which is 1. More precisely, every permutation matrix is a product of elementary row interchange matrices. Using this concept, we can find six 3x3-permutation matrices and here we give one of them as follows:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

To proceed in this study, we recall the following information about various codes from the literature.

B. Double weight code (DW):

The double weight code (DW) given in [12] and the code represented $K \times C$ matrix, where K is the number of users and C is the minimum code length. A 2×3-DW matrix represented as

$$H_{M=1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{1}$$

The method given in this code is by considering the following technique. For each three column, the combination sequence is 1, 2, 1 to keep the maximum cross correlation one. To increase the number of users [see 13, 14] the number of rows and columns are doubled.

C. *Modified Double Weight (MDW) code*

The modified double weight (MDW) code, which has weight, more than two (multiple of two) to increase the signal to noise ratio (SNR) and all other properties of DW and MDW are same. The modified weight code offered by using $K \times C$ matrix, where K represents the number of users and C is the code length. ZCC code with weight 1 (for $W=1$) shown in (3)

$$Z_{M=1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2}$$

It is clear that Z_1 has no overlapping of '1' for both users. To increase the number of users and code length, a mapping technique is used as follows:

$$Z_{M=2} = \begin{bmatrix} 0 & Z_1 \\ Z_1 & 0 \end{bmatrix} \tag{3}$$

Recently the zero cross correlation code developed in [14], called ZCC represented a matrix of dimension $K \times C$ where K is the number of users (rows) and C is minimum code length (columns) .In this study, the authors considered the matrix

$$Z_W = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{4}$$

Where [A] consists of w replication of matrix (Z_w =1) as shown in equation (8) ,[B] contains [2, W (K-2)] matrix of zero,[C] contains [(K-2), $W \times 2$] matrix of zero and [D] contains [(K-2), $W \times$(K-2)] matrix of 90° rotation diagonal pattern [(K-2)$\times$(K-2)] with w replication of each matrix.

Very recently, Nisar [15] developed a new zero cross correlation code with numerous advantages. The new ZCC constructed by considering a type of anti-diagonal-identity-column-block matrices $N \times L$ where N is the number of users and L (columns) represents the minimum code length. A basic new ZCC [15] for $W = 1$ shown in equation (5)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{5}$$

To increase number of users and code weight, the author adopted the following mapping technique.

$$\left[\begin{array}{c:cccc:cc:c} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Which is the code pattern for $W = 2$.

Motivated from the above code sequences, in this paper we develop a new method to construct zero cross correlation code named as permutation Matrix Zero Cross Correlation (PM-ZCC) code and compare the advantages with other existing codes in the literature.

II. NEW PM-ZCC CODE CONSTRUCTION

A new zero cross correlation code PM-ZCC denoted by the matrix of order $N \times L$, where N is the number of users and L is the minimum code length. A basic PM-ZCC code for $W = 1$ and, $N = 2$ denoted by $P_{W,N}$ shown in equation (11)

$$P_{1,2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

Also, we have $P_{1,3}$ and $P_{1,4}$ as follows:

$$P_{1,3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{7}$$

$$P_{1,4} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{8}$$

From the definition of permutation matrix, it is easy to obtain one of the 5x5-permutation matrix as:

$$P_{1,5} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{9}$$

Notice that the number of users increases with the increasing code length. To increase the code weight, we use the following matrix pattern:

For $W = 2$ and $N = 4$

$$P_{2,,4} = \left[\begin{array}{cccc:cccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{10}$$

For $W = 3$

(10)

$$P_{3,,4} = \left[\begin{array}{cccc:cccc:cccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{11}$$

More generally, if $W = N$ then we have

$$P_{n,N} = n \times N\ . \tag{12}$$

From the above matrix pattern, we obtained the equation of code length L as:

$$L = N \times W \tag{13}$$

Using (12) and (13) one can easily construct the code for $W = 2$ and $N = 5$:

$$P_{2,5} = \begin{bmatrix} 0 & 1 & 0 & 0 & \vdots & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \vdots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \vdots & 0 & 0 & 1 & 0 \end{bmatrix} \quad (14)$$

The block diagram of PMC matrix pattern given in Fig. 1:

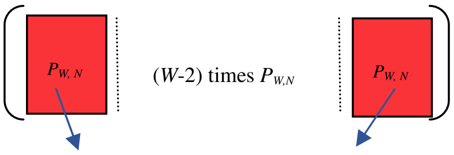

Fig.1: PM-ZCC block diagram representations.

III. Code Comparison

The most important parameter that needs take care while designing a code in Spectral Amplitude Coding OCDMA is the code length. For the given number of users and code weight, a code with minimum code length is preferable. This is because of the fact that longer code length will lead to larger bandwidth consumptions of each code. Apart from minimizing the capacity that will dealt by each system, larger bandwidth would contribute to higher noise level. Another disadvantageous of larger code length is either very wide band sources or very narrow filter bandwidths are required.

The comparison of code properties of OOC [5], Prime Code [11], ZCC [6], NZCC [9] and new PM-ZCC codes are listed in Table 1. The suggested PM-ZCC code family supports practical code length that is neither too short nor too long which gives good property of encoder-decoder design structure.

Table 1: Code Comparison in term of code length for same number of subscribers ($N = 30$)

Codes	No. of User (N)	Weight (W)	Code Length (L)
OOC [5]	30	4	364
Prime Code [12]	30	6	1296
ZCC [6]	30	4	120
NZCC[11]	30	4	120
PM-ZCC	30	4	120

IV. Concluding Remark

A new zero cross correlation code named PM-ZCC for spectral-amplitude encoding OCDMA system is successfully developed. The PM-ZCC code have numerous advantages such as: (1) simplicity in code construction; (2) Can choose any positive integer number of code weight; (3) Code length shorter than OOC and Prime codes and it compete with ZCC and NZCC codes.

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