International Journal of Open Information Technologies ISSN: 2307-8162 vol. 14, no. 2, 2026

On the System of Reaction-Diffusion Equations in a Limited Region

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***Abstract* — This paper addresses problems of dynamics and long-term behavior of replicator (nonlinear) systems of partial differential equations. The primary focus is on the influence of the spatial factor on the behavior of distributed systems described by partial differential equations. A general problem formulation with Neumann, Dirichlet, and Robin boundary conditions is considered, and both spatially homogeneous and inhomogeneous stationary equilibrium states are analyzed. The stability of these states is investigated using spectral analysis and the energy method, including generalizations for various types of boundary conditions. The paper demonstrates that for sufficiently large diffusion coefficients, solutions tend to a stationary regime, with Dirichlet and Robin conditions enhancing stability compared to Neumann conditions. Examples, such as the Fisher-Kolmogorov equation and a two-component system, are provided to illustrate the application of the proposed methods. The results emphasize the importance of accounting for boundary conditions and diffusion in predicting the long-term behavior of reaction-diffusion systems.**

***Keywords* — Reaction-diffusion, Nonlinear systems, Partial derivatives, Spatial stability, Boundary conditions, Neumann conditions, Dirichlet conditions, Robin conditions, Stationary equilibrium, Spectral analysis, Energy method, Sobolev space, Fisher–Kolmogorov equation, Diffusion flows, Asymptotic stability.**

I. INTRODUCTION

Let us consider the general formulation of such a problem. Let a system of differential equations of the form be given in a bounded domain $\Omega \subset R^m$:

$$\frac{\partial u}{\partial t} = \sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right) + f(u), x \in \Omega, t>0 \quad (1)$$

where $u = (u_1,\ldots,u_n)^T, f = (f_1,\ldots,f_n)^T, x = (x_1,\ldots,x_m)$

Here $A(x) = \left(a_{ij}(x)\right)$, i,j=1,…,m is a symmetric matrix with real positive eigenvalues.

At the initial time t=0, the initial conditions are given:

$$u(x,0) = u_0(x), x \in \Omega \quad (2)$$

and on the boundary $\partial\Omega$ of the domain Ω зhomogeneous boundary conditions of the 2nd kind (Neumann conditions) are specified:

$$\frac{\partial u}{\partial \nu} = 0, x \in \partial\Omega, t>0 \quad (3)$$

where $\partial\nu$ is the unit outward normal to the boundary $\partial\Omega$.

The system (1)–(3) is closed because the fluxes of the reacting components through the domain boundary are zero.

In the literature, such systems are called "reaction-diffusion" systems.

Here the vector function $f(u)$ determines the reaction of the components, which is described by the dynamical system:

$$\frac{du}{dt} = f(u)$$

The matrix of coefficients $A(x)$ describes the diffusion fluxes arising in the domain Ω.

In the classical case, diagonal matrices $A(x)$ are considered. In this case, the so-called cross-diffusion fluxes are not taken into account, where the diffusion flux of one component of the system influences the dynamics of another component.

In this work, we will subsequently consider weak solutions [2] of problem (1)–(3), which are elements (for each fixed t) of the Sobolev space $W^{1,2}(\Omega)$ with the norm

$$\|u\|_{W^{1,2}(\Omega)} = \left(\int (|u^2| + |\nabla u^2|)\, dx\right)^{1/2}$$

and for any t⩾0 they represent smooth functions in the variable t.

The class of such functions satisfying the above requirements will henceforth be denoted as V.

II. STATIONARY EQUILIBRIUM STATES

A. Definition 1

A vector function $\bar{u}(x) \in V$ such that:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \bar{u}}{\partial x_j}\right) + f(\bar{u}) = 0, x \in \Omega \quad (4)$$

is called a stationary equilibrium position (or steady state) of system (1)–(3).

If the equilibrium position is $\bar{u}(x) \neq$ const, then it is called spatially inhomogeneous. The problem of finding spatially inhomogeneous equilibria is very complex. We will assume that $\bar{u}(x)$ s a spatially homogeneous equilibrium position, i.e., there exists a solution to the problem:

$$f(\bar{u}) = 0 \quad (5)$$

The study of such equilibrium positions provides information about the limiting state of system (1)–(3) as $t \to \infty$. As in the case of dynamical systems, we introduce an analogue of the concept of Lyapunov stability for stationary equilibrium positions.

B. Definition 2

The equilibrium position $\bar{u}(x)$ of system (1)–(3) is called Lyapunov stable if for $\forall\varepsilon>0$ there exists $\delta>0$ такое, что such that $u_0 \in V$ for all solutions $u(t,x)$ cof system (1)–(3) with initial data u_0 satisfying:

$$\|u_0 - \bar{u}\|_V < \delta, \text{forall } t \geq 0, \|u(t,.) - \bar{u}\|_V < \varepsilon$$

Manuscript received December 9, 2025.
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If, in addition, the following condition holds:

$$\|u(t,.) - \bar{u}\|_V \to 0, \text{for } t \to \infty$$

then the equilibrium position is called asymptotically stable.

Let be $\bar{u}(x)$ a spatially homogeneous equilibrium position of system (1)–(3).

Consider the Jacobian matrix of the vector function f:

$$J = \frac{\partial f}{\partial u}(\bar{u})$$

The investigation of the stability of the equilibrium position can be carried out using an analogue of the Lyapunov–Poincaré theorem on stability with respect to first approximation [4]. It then reduces to studying the spectrum of the following eigenvalue problem:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \phi}{\partial x_j}\right) + J\phi = \lambda\phi, x \in \Omega \quad (6)$$

with boundary conditions

$$\frac{\partial \phi}{\partial \nu} = 0, x \in \partial\Omega$$

The corresponding eigenvalues form a non-decreasing sequence:

$$\lambda_1 \leqslant \lambda_2 \leqslant \dots$$

If the condition holds for all eigenvalues of problem (6) $\mathrm{Re}(\lambda_k)<0, k==1,2,\dots$, then the equilibrium position is asymptotically stable. The precise formulation of this theorem can be found in [5].

Consider the linear transformation $v = P^{-1}\phi$, where P — is a matrix such that $P^T JP == \Lambda, P^T$ is the transposed matrix.

Taking this transformation into account, the spectral problem (6) takes the form:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial v}{\partial x_j}\right) + \Lambda v = \lambda v, x \in \Omega \quad (7)$$

where $\Lambda = \mathrm{diag}(\gamma_1,\dots,\gamma_n)$.

We will seek a solution to problem (7) in the form:

$$v(x) = \sum c_k \psi_k(x), c_k \in R^n \quad (8)$$

where ψ_k,quadk=1,2,... are the eigenfunctions of the following boundary eigenvalue problem:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \psi}{\partial x_j}\right) = -\mu\psi, x \in \Omega \quad (9)$$

with boundary conditions

$$\frac{\partial \psi}{\partial \nu} = 0, x \in \partial\Omega.$$

It is known [3,6,7] that problem (9) has a biorthonormal system of eigenfunctions ψ_k, which form a complete system in the space $L^2(\Omega)$, and the following holds:

$$(\psi_k, \psi_l)_{L^2(\Omega)} = \int \psi_k \psi_l dx = \delta_{kl}, k,l=1,2,\dots \quad (10)$$

where δ_{kl} is the Kronecker delta.

The corresponding eigenvalues form a non-decreasing sequence $0=\mu_1 \leqslant \mu_2 \leqslant \dots$

Taking the representation (8) into account, the original problem takes the form:

$$\sum(-\mu_k c_k \psi_k + \Lambda c_k \psi_k) = \lambda \sum c_k \psi_k$$

If we multiply this equality scalarly in the space $L^2(\Omega)$ by the functions ψ_l , where l= 1,2,... , и воспользоваться соотношением (10), то получим матричные равенства для вектand use relation (10), we obtain matrix equalities for the vectors c_k in the form of eigenvalue problems:

$$(\Lambda - \mu_k I)c_k = \lambda c_k, k=1,2,\dots \quad (11)$$

Thus, the problem of finding the eigenvalues of the continuum system (6) reduces to the algebraic problem of finding the eigenvalues of a countable sequence of matrices of the form:

$$\lambda_{k,j} = \gamma_j - \mu_k, k=1,2,\dots,j=1,\dots,n \quad (12)$$

If for all eigenvalues of problem (11) the condition:

$$\mathrm{Re}(\lambda_{k,j})<0, k=1,2,\dots,j=1,\dots,n$$

holds, then the spatially homogeneous equilibrium position $\bar{u}$ of system (1)–(3) is stable.

If, however, this condition fails for at least one value k,j then the equilibrium position is unstable.

III. EXAMPLES

Let's consider several examples of applying the formulated results to specific problems.

A. *Example 1*

Let us write the Fisher–Kolmogorov equation on the interval $\Omega = (0,\pi)$ with homogeneous Neumann boundary conditions:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + u(1-u), x \in (0,\pi), t>0 \quad (13)$$

$$\frac{\partial u}{\partial t}(0,t) = \frac{\partial u}{\partial t}(\pi,t) = 0, t>0 \quad (14)$$

This equation has two spatially homogeneous equilibrium states, u=0 and u=1 . The second equilibrium state is determined by the eigenfunctions and eigenvalues of problem (9):

$$\psi_k(x) = \sqrt{\frac{2}{\pi}}\cos(kx), \mu_k = k^2 D, k=0,1,2,\dots$$

Equality (12) takes the form:

$$\lambda_k = -1 - k^2 D, k=0,1,2,\dots$$

Consequently, the equilibrium state u=1 is asymptotically stable [8,9].

In the case u=0, it follows from equality (12) that $\lambda_k = 1 - k^2 D$, k=0,1,2,.... Thus, the equilibrium state is unstable, since $\lambda_0 = 1 > 0$.

B. *Example 2*

Let us now consider another example of a reaction-diffusion system. We will investigate the influence of diffusion on the behavior of a closed reaction-diffusion system of general form for $t \to \infty$. We will focus [11] on the case n=2.

So, let us consider a system of the form:

$$\frac{\partial u_1}{\partial t} = d_1 \Delta u_1 + f_1(u_1, u_2), \quad \frac{\partial u_2}{\partial t} = d_2 \Delta u_2 + f_2(u_1, u_2), \quad x \in \Omega, t>0 \quad (15)$$

where $u = (u_1, u_2)^T$, $f = (f_1, f_2)^T$. Here d_1 and d_2 are the diffusion coefficients, and is the Laplace operator.

The functions u_1 and u_2 satisfy Neumann boundary conditions (the case of a closed system):

$$\frac{\partial u_1}{\partial \nu} = \frac{\partial u_2}{\partial \nu} = 0, \quad x \in \partial\Omega, t>0 \quad (16)$$

on the boundary $\partial\Omega$ of the bounded closed domain Ω and homogeneous initial conditions:

$$u_1(x,0) = u_{1,0}(x), \quad u_2(x,0) = u_{2,0}(x), \quad x \in \Omega \quad (17)$$

For definiteness, we will assume that the domain Ω is a square:

$$\Omega = [0,\pi] \cdot [0,\pi]$$

The vector function $f(u)$ determines the reaction of the components of the system (15)–(17), which is described by the dynamical system:

$$\frac{du}{dt} = f(u)$$

The matrix $A(x) = \mathrm{diag}(d_1, d_2)$ describes the diffusion fluxes arising in the domain Ω.

We will consider the solutions of the system (15)–(17) in the Sobolev space $W^{1,2}(\Omega)$.

To study the behavior of the solutions of the system (15)–(17) for $t \to \infty$, we will use the energy (variational) method [1,3,5].

To do this, we introduce a (variational) function of time:

$$E(t) = \frac{1}{2}\int (u_1^2 + u_2^2)\,dx \quad (18)$$

which plays the role of the system's energy.

Let us compute the derivative of the function (18) taking into account (15)–(17). We obtain that:

$$\frac{dE}{dt} = -\int (d_1|\nabla u_1|^2 + d_2|\nabla u_2|^2)\,dx + \int \left(u_1 f_1(u_1,u_2) + u_2 f_2(u_1,u_2)\right) dx \quad (19)$$

Formula (19) can be represented in the form:

$$\frac{dE}{dt} = I_1 + I_2$$

where:

$$I_1 = -\int (d_1|\nabla u_1|^2 + d_2|\nabla u_2|^2)\,dx \quad (20)$$

$$I_2 = \int \left(u_1 f_1(u_1,u_2) + u_2 f_2(u_1,u_2)\right) dx \quad (21)$$

Let us introduce the notation:

$$\alpha = \min(d_1, d_2), \quad \beta = \sup_{u\in R^2}\left|\frac{\partial f_1}{\partial u_1} + \frac{\partial f_2}{\partial u_2}\right| \quad (22)$$

C. Theorem 1

If $\alpha > \beta$, then $\frac{dE}{dt} \leqslant 0$ nd therefore all partial derivatives $\frac{\partial u_1}{\partial x_i}, \frac{\partial u_2}{\partial x_i}$, $i=1,2$, tend to zero as $t \to \infty$, and the solution itself tends to a constant (a stationary regime).

To prove the theorem, it is sufficient to consider the integral (20). Here, after integration by parts [12], taking into account the boundary conditions, we rewrite (20) in the form:

$$I_1 = -\int (d_1|\nabla u_1|^2 + d_2|\nabla u_2|^2)\,dx$$

Then, considering the notation (22) and by virtue of the Cauchy–Bunyakovsky inequality, we have:

$$I_2 \leqslant \beta \int (u_1^2 + u_2^2)\,dx \quad (23)$$

Since $\alpha = \min(d_1, d_2)$, for the first integral on the right side of the last inequality, Friedrichs' inequality holds, and therefore the following inequalities are valid:

$$\int |\nabla u_i|^2\,dx \geqslant C_F \int u_i^2\,dx, \quad i=1,2 \quad (24)$$

where C_F is the Friedrichs constant.

Using (23) and (24), we obtain the following estimate:

$$\frac{dE}{dt} \leqslant -\alpha C_F \int (u_1^2 + u_2^2)\,dx + \beta \int (u_1^2 + u_2^2)\,dx$$

From the last inequality, we conclude that $\alpha > \beta$, then $\frac{dE}{dt} \leqslant 0$.

Since $E(t) \geqslant 0$, and $E(t) \to E_\infty \geqslant 0$ then $\int (u_1^2 + u_2^2)\,dx \to 0$ as $t \to \infty$.

Applying similar reasoning to the second integral (21), we obtain the relation $\int |\nabla u_i|^2\,dx \to 0$, and consequently $\frac{\partial u_i}{\partial x_j} \to 0$, $i=1,2$, $j=1,2$.

Therefore, it is obvious that all partial derivatives $\frac{\partial u_1}{\partial x_i}, \frac{\partial u_2}{\partial x_i}$, $i=1,2$, tend to zero as $t \to \infty$, and the solution itself tends to a stationary state. The theorem is proven.

IV. GENERALIZATION TO BOUNDARY CONDITIONS OF THE FIRST AND THIRD KIND

In this section, we generalize the obtained results to boundary conditions of the first kind (Dirichlet conditions) and the third kind (Robin conditions), investigating their influence on the dynamics and stability of the system (1)–(3).

A. Dirichlet Boundary Conditions (First Kind)

Dirichlet boundary conditions are specified as:

$$u(x,t) = g(x), x \in \partial\Omega, t>0 \quad (25)$$

where $g(x)$ is a given function on the boundary $\partial\Omega$. For simplicity, we will assume $g(x)=0$, which corresponds to homogeneous Dirichlet conditions:

$$u(x,t)=0, x \in \partial\Omega, t>0 \quad (26)$$

These conditions model a situation where the values of the components u are fixed on the boundary.

To investigate the stability of the spatially homogeneous equilibrium $\bar{u}$, which satisfies $f(\bar{u})=0$, we consider a spectral problem analogous to (6), but with Dirichlet boundary conditions:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \phi}{\partial x_j}\right) + J\phi = \lambda\phi, x \in \Omega \quad (27)$$

$$\phi=0, x \in \partial\Omega \quad (28)$$

where $J = \frac{\partial f}{\partial u}(\bar{u})$ is the Jacobian matrix. The eigenfunctions ψ_k of the boundary value problem

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \psi}{\partial x_j}\right) = -\mu\psi, x \in \Omega, \quad \psi=0, x \in \partial\Omega \quad (29)$$

form a biorthonormal system in $L^2(\Omega)$, and the eigenvalues μ_k satisfy $0<\mu_1 \leqslant \mu_2 \leqslant \ldots$, in contrast to the Neumann case where $\mu_1=0$. The eigenvalues of the system are determined by

$$\lambda_{k,j} = \gamma_j - \mu_k, k=1,2,\ldots, j=1,\ldots,n \quad (30)$$

where γ_j are the eigenvalues of the matrix Λ obtained from J via a transformation $P^T J P = \Lambda$. The equilibrium is asymptotically stable if $\mathrm{Re}(\lambda_{k,j})<0$ for all k,j. Since $\mu_k>0$, Dirichlet conditions can promote greater stability compared to Neumann conditions, as $-\mu_k$ вcontributes a negative term to $\lambda_{k,j}$.

To analyze the behavior of solutions as $t \to \infty$ we apply the energy method, similar to Section 2.2. The system's energy is defined as in (18). However, when computing the derivative $\frac{dE}{dt}$ the boundary terms vanish due to $u=0$ on $\partial\Omega$, and formula (19) remains valid. The Friedrichs inequality for Dirichlet conditions provides a stronger estimate:

$$\int |\nabla u_i|^2\,dx \geqslant C'_F \int u_i^2\,dx, i=1,2 \quad (31)$$

where $C_F' > C_F$ due to the absence of a zero eigenvalue. Thus, if $\alpha > \beta$, as defined in (22), then $\frac{dE}{dt} \leqslant 0$, and the solution tends to a stationary regime, with Dirichlet conditions enhancing the convergence to a constant.

B. Robin Boundary Conditions (Third Kind)

Robin boundary conditions (third kind) are specified as:

$$\frac{\partial u}{\partial \nu} + \sigma u = 0, x \in \partial\Omega, t>0 \quad (32)$$

where $\sigma \geqslant 0$ s a parameter describing the balance between the flux and the function value at the boundary. These conditions model partial permeability of the boundary, where $\sigma=0$ corresponds to Neumann conditions, while $\sigma \to \infty$ approximates Dirichlet conditions.

In this case, the spectral problem for stability takes the form:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \phi}{\partial x_j}\right) + J\phi = \lambda\phi, x \in \Omega \quad (33)$$

$$\frac{\partial \phi}{\partial \nu + \sigma}\phi = 0, x \in \partial\Omega \quad (34)$$

The boundary value problem for the eigenfunctions becomes:

$$\sum \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial \psi}{\partial x_j}\right) = -\mu\psi, x \in \Omega$$

$$\frac{\partial \psi}{\partial \nu} + \sigma\psi = 0, x \in \partial\Omega \quad (35)$$

The eigenvalues μ_k satisfy $0<\mu_1 \leqslant \mu_2 \leqslant \dots$, where $\mu_1=0$ only when $\sigma=0$. For $\sigma>0$ all $\mu_k>0$, which enhances the negative contribution to $\lambda_{k,j} = \gamma_j - \mu_k$. Stability is determined similarly: the equilibrium is asymptotically stable if $\mathrm{Re}(\lambda_{k,j})<0$ for all k,j. Robin conditions with $\sigma>0$ promote stability, especially for large σ, making the system's behavior approach that of the Dirichlet case.

When computing $\frac{dE}{dt}$ for Robin conditions, the boundary terms provide an additional contribution:

$$\frac{dE}{dt} = -\int (d_1|\nabla u_1|^2 + d_2|\nabla u_2|^2)\,dx -$$
$$-\int \sigma\,(u_1^2 + u_2^2)dS + \int (u_1 f_1 + u_2 f_2)\,dx \quad (36)$$

The boundary integral $\int \sigma\,(u_1^2 + u_2^2)dS \geqslant 0$ enhances energy dissipation since $\sigma \geqslant 0$. Using the Friedrichs inequality adapted for Robin conditions, we find that if $\alpha > \beta$, then $\frac{dE}{dt} \leqslant 0$, and the solution tends to a stationary regime. For $\sigma>0$ the additional boundary term accelerates convergence, making Robin conditions intermediate between Neumann and Dirichlet conditions.

V. CONCLUSION

The proposed method [3,12] makes it possible to maintain stability for sufficiently large diffusion coefficients. This type of stability is commonly referred to as spatial-diffusive stability, even in the case of a system that would be unstable in the absence of diffusion (i.e., when $t \to \infty$).

It is evident that the obtained result extends to the case of system (1)–(3). However, this result cannot always be directly applied in specific cases [16,17], as the calculation of the constant β depends on having a priori knowledge about the solution of system (15)–(17) (or system (1)–(3)) and its derivatives.

Boundary conditions of the first and third kind significantly influence the behavior of reaction-diffusion systems [13,14,15,18]. Dirichlet conditions, which fix zero values at the boundary, enhance stability and convergence to a stationary regime due to positive eigenvalues μ_k. Robin conditions, depending on the parameter σ, provide a flexible transition between Neumann [19] and Dirichlet [20] conditions, and for $\sigma>0$ they also promote stability. The energy method confirms that for sufficiently large diffusion coefficients $\alpha > \beta$, solutions tend to a spatially homogeneous stationary state, with Dirichlet and Robin conditions [21] (for $\sigma>0$) accelerating this process compared to Neumann conditions.

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