# Limits of the quasi-static approach for calculating radiated emission from Power Electronics Converters

Moez Youssef, Yousef Ibrahim Daradkeh

*Abstract*—With the increase of switching frequencies in power electronics converters, the range validity of circuit models must be more and more larger. A useful method for interconnect modeling is the Partial Element Equivalent Circuit technique. The model given by this technique can be combined with other circuit models (like transistors) into an input circuit for a circuit simulator. This technique can be used when the quasi-static hypothesis is verified. In the other case, this technique can be extended in order to include propagation and retardation. In this paper, we will try to carry out the limits of the quasi-static approach.

### *Index Terms*—Power Electronics Converters, Electromagnetic Compatibility, Circuit simulator, PEEC technique.

## I. INTRODUCTION

In the field of power electronics, the potential for unwanted emissions at high frequencies increases as advances in switched mode power converter technology push the switching frequency higher and higher.

A traditional approach used to control EMI in switchingmode power supplies mainly consists in performing EMC test prototypes at the end of the productive cycle and modifying the layout if the expected EMC performances are not matched. The risk of this approach is a dangerous delay in placing the product on the market since identification of the causes of failures, modification and successfully retesting of the product are required.

An alternative way is to develop CAD tool in order to predict the current distribution and the radiated electromagnetic field. The two principal problems are the modeling of power electronics semiconductor devices and interconnections. This paper deals with the second subject.

The purpose that consists in developing a CAD tool motivated the translation of the electromagnetic interconnect problem to the circuit domain.

The most common approach for the circuit modeling of three dimensional geometries is the partial element equivalent circuit approach (PEEC) [1,2]. One of the useful aspects of the PEEC approach is its generality. The models are applicable both in the time and the frequency domain.

Further, calculations for the partial capacitances or partial inductances can be performed independently of the actual

circuit domain computations.

Also, by its structure, the PEEC approach permits to identify the parts of the circuit which involve important

## radiations.

The PEEC approach is valid for quasi-static problems. PEEC model which include retarded effect is called rPEEC.

This extension opens a world of new applications to circuit models. Example is antenna modeling.

In section II, we give a review of the general method for calculating radiated field from the current distribution.

In section III and IV, we explain how to obtain both the PEEC model and the model including retardation (rPEEC).

In section V, we show the limits of the quasi-static approach and in section VI we draw conclusions.

## II. FIELD COMPUTATION

In order to calculate magnetic and electric field, we use the potentials  $\vec{A}$  and V defined by:

$$\vec{H} = \frac{1}{\mu_0} \operatorname{rot} \vec{A} \tag{1}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - grad \ V \tag{2}$$

These potentials are related by the Lorentz condition:

$$div\,\vec{A} + \mu_0 \varepsilon \frac{\partial V}{\partial t} = 0 \tag{3}$$

Then, the knowledge of the magnetic vector potential  $\vec{A}$  suffices to calculate  $\vec{E}$  and  $\vec{H}$ .[1,2,11]

If we combine Maxwell's equations with the previous relations, we find an equation with the vector potential  $\vec{A}$  as unknown:

$$\nabla^2 \vec{A} - \mu_0 \varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$
(4)

The solution of (4) is:

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}\left(r', t - \frac{|r - r'|}{c}\right)}{|r - r'|} dv'$$
(5)

In the frequency domain, we have:

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r') e^{j\omega\left(t - \frac{|r-r'|}{c}\right)}}{|r-r'|} dv'$$
(6)

Where r' is the position of the source point and r is the position of the observation point.

We see that in order to have the vector potential, we must know the current distribution.

This work was supported in part by the Deanship of Scientific Research, Prince Sattam bin Abdulaziz University, Saudi Arabia. Project number is 2014/1/863.

Moez Youssef is with Prince Sattam bin Abdulaziz University (email: ymoez@yahoo.fr )

Yousef Ibrahim Daradkeh is with Prince Sattam bin Abdulaziz University (email: daradkehy@yahoo.ca)

When  $|r-r'| \ll \lambda$ , retardation can be neglected, otherwise if  $L \ll \lambda$  (L represents dimension of the conductor), we can consider that the current is constant along the conductor, in this case the expression of the potential vector is reduced to:

$$\vec{A}(r) = \frac{\mu_0 i}{4\pi} \int \frac{dv'}{|r-r'|}$$
(7)

This expression can be calculated analytically [13].

If  $L \approx \lambda$ , the conductor must be subdivided into cells with different currents. In the two next sections, we will see how to find this current, as a first step when retardation is neglected and as a second step in the general case.

### III. PEEC MODEL

The unknowns in a multi-conductor system are the charges on the surfaces and the current densities within the conductors.

The integral equation to resolve is based on two relations:

\* The Ohm's law, that gives the total electric field:

$$\vec{E} = \frac{J}{\sigma} \tag{8}$$

\* The total field can be subdivided into two parts:

$$\vec{E}_T = \vec{E}_0 + \vec{E}' \tag{9}$$

Where  $E_0$  is the applied field and E' is the induced field due to the charges and the currents of the conductor itself:

$$\vec{E}' = -\frac{\partial A}{\partial t} - grad \ V \tag{10}$$

The vector potential is given by (6), the scalar potential has a similar expression:

$$V(r) = \frac{1}{4\pi\varepsilon} \int \frac{q(r') e^{j\omega\left(t - \frac{|r-r'|}{c}\right)}}{|r-r'|} dv'$$
(11)

If we introduce (5), (8) and (10) in (9), we find:

$$\vec{E}_0(r) = \frac{\vec{J}(r)}{\sigma} + j\omega \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r')e^{j\omega\left(t - \frac{|r-r'|}{c}\right)}}{|r-r'|} dv' + gradV$$
(12)

As a first step, we restrict ourselves to rectilinear geometries. The conductor must be subdivided into inductive and capacitive cells, the two subdivisions are displaced by  $\frac{\Delta}{2}$ . The capacitive cells C<sub>k</sub> have a potential V<sub>k</sub> and a charge q<sub>k</sub>. The inductive cells  $\ell_k$  have uniform current i<sub>k</sub>. (Figure 1). [1]



Figure 1. Conductor subdivision into inductive and capacitive cells

For this section retardation is neglected.

Because (12) holds everywhere on the conductor, we can integrate it over an inductive cell  $\ell_k$ .

The term on the left-hand side of (12) is:

$$\int_{\ell_k} E_0(x) ds_k = w V_{0k} \tag{13}$$

Where w is the width of the conductor,  $V_{0k}$  is the applied voltage on the cell  $\ell_k$ .

The first term on the right-hand side is:

$$\int_{\mathscr{V}_k} \frac{J(x)}{\sigma} ds_k = i_k \frac{\Delta}{\sigma} = w.R.i_k \tag{14}$$

Where  $\Delta$  is the length of an inductive cell, and R its resistance.

The second term on the right-hand side is:

$$j\omega \int_{\ell_k} A(x) ds_k = j\omega \frac{\mu}{4\pi w} \left[ i_k \cdot \int_{\ell_k \ell_k} \frac{ds_k ds'_k}{|x-x'|} + \sum_{k' \neq k} i_k \cdot \int_{\ell_k \ell_k} \frac{ds_k ds_k'}{|x-x'|} \right] (15)$$

Using the definition of partial mutual inductances, the relation can be written as:

$$\int_{\ell_k} \mathcal{A}(x) ds_k = w \left[ j \omega L_{pk} \, i_k + \sum_{k' \neq k} j \omega \, L_{pkk'} \, i_{k'} \right]$$
(16)

Where  $L_p$  are self and mutual partial inductances.

The third term on the right-side is:

$$\int_{\ell_k} \frac{\partial V(x)}{\partial x} ds_k = w \left[ V_{k+1} - V_k \right]$$
(17)

The potential can be written as:

1

9

$$V_{k} = \sum_{k'} P_{kk'} \cdot q_{k'}$$
(18)

Where  $q_{k'}$  is the total charge in the k' capacitive cell and:

$$P_{kk'} = \frac{1}{4\pi\varepsilon S_k S_{k'}} \int_{C_k} \int_{C_{k'}} \frac{ds_k ds_{k'}}{|x - x'|}$$
(19)

are coefficients of potential. Then, we have:

$$\begin{cases} V = P.q \\ q = c.V \\ c = P^{-1} \end{cases}$$
(20)

In order to have an equivalent circuit, the expression of the charges must include potential difference:

$$V_{k} = \left(\sum_{k'} c_{kk'}\right) \cdot V_{k} - \sum_{k'} c_{kk'} \cdot V_{kk'}$$
 (21)

Then, capacitance matrix can be deduced by [3,4,6,7]:

$$\begin{cases} C_{kk} = \left(\sum_{k'} c_{kk'}\right) \\ C_{kk'} = -c_{kk'} & \text{for } k \neq k' \end{cases}$$
(22)

For a rectilinear conductor, we obtain an equivalent circuit as represented in figure 2.



In parallel with the circuit approach, we can express the

(29)

problem by a matrix equation with the current distribution as an unknown. The general equations can be written as:

$$e_{k} = R.i_{k} + j\omega \sum_{k'} Lp_{kk'} \cdot i_{k'} + V_{k+1} - V_{k}$$
(23)

$$V_k = \sum_{k'} P_{kk'} \cdot q_{k'} \tag{24}$$

In order to have a system with the current distribution, we use the continuity equation:

$$\frac{\partial i}{\partial x} + j \,\omega q = 0 \tag{25}$$

That can be written as:

$$q_k = -\frac{1}{j\omega} \frac{I_k - I_{k-1}}{\Delta}$$
(26)

Then:

$$V_{k} = \frac{1}{j\omega} \sum_{k'} \left( P_{k,k'+1} - P_{k,k'} \right) \cdot i_{k'}$$
(27)

The equation (26) becomes:

$$e_{k} = R.i_{k} + j\omega \sum_{k'} Lp_{kk'} \cdot i_{k'} + \frac{1}{j\omega} \sum_{k'} i_{k'} \cdot \left( P_{k+1,k'+1} - P_{k,k'+1} - P_{k+1,k'} + P_{k,k'} \right)$$
(28)

This can be written in a matrix form:

Z.I = E

Where:

$$Z_{kk'} = \delta(k, k') \cdot R + j \omega L p_{kk'} + \frac{1}{j\omega} (P_{k+1,k'+1} - P_{k,k'+1} - P_{k+1,k'} + P_{k,k'})$$
(30)

And E is the matrix of applied voltage. We can treat either the case of local source or the case of distributed source.

## IV. RPEEC MODEL

In this section, we will take into consideration the retardation effect. The inductive term becomes:

$$j\omega \int_{\ell_{k}} A(x)ds_{k} = j\omega \frac{\mu_{0}}{4\pi w} \sum_{k'} i_{k'} \cdot \int_{\ell_{k}\ell_{k'}} \frac{e^{-j\beta|x-x'|}ds_{k} ds_{k} '}{|x-x'|}$$
$$= \sum_{k'} i_{k'} \cdot \left[ \frac{j\omega\mu_{0}}{4\pi w} \int_{\ell_{k}\ell_{k'}} \frac{\cos(\beta|x-x'|)ds_{k} ds_{k} '}{|x-x'|} + \frac{\omega\mu_{0}}{4\pi w} \int_{\ell_{k}\ell_{k'}} \frac{\sin(\beta|x-x'|)ds_{k} ds_{k} '}{|x-x'|} \right]$$
(31)

Comparing to the last section, we see that the inductive term contains a resistive term:

$$j\omega \int_{\ell_k} A(x) ds_k = w \sum_{k'} \left( Rr_{kk'} + j \omega Lp_{kk'} \right) \cdot i_{k'}$$
(32)

Where  $Rr_{kk}$ , are the self and mutual partial radiation resistance.

#### Examples of calculation of partial radiation resistance:

If we take two filiform parallel conductors, the partial radiation resistance between them is:

$$Rr_{12} = \frac{\omega \mu_{0}}{4\pi w} \int_{\ell_{k}\ell_{k'}} \frac{\sin(\beta |x-x'|) dx_{2} dx_{1}}{|x-x'|}$$
(33)

When the lengths of the conductors and the distance between them are negligible comparing to wavelength, the relation (36) can be reduced to:

$$Rr_{12} \approx \frac{\omega \mu_0}{4\pi w} \int_{\ell_k \ell_k'} \beta \, dx_2 \, dx_1 \approx 30 \, \beta^2 \, \ell_1 \, \ell_2 \tag{34}$$

For example, the partial radiation resistance of a 10cm length conductor at f=100MHz is  $Rr = 1.314\Omega$ .

We see that the mutual partial radiation resistance is independent of the distance between the conductors since this distance is negligible comparing to the wavelength.

Otherwise, the partial radiation resistance of a rectangular conductor varies in  $f^4$ :

$$Rr_{rectan\,gular} \approx 20 \,\beta^4 \,\ell_1^2 \,\ell_2^2 \tag{35}$$

For example, for a rectangular conductor 10x10cm, the radiation resistance is:  $Rr_{rectan gular} \approx 40m\Omega$ .

As a conclusion, for the inductive model, we see that the retardation involve a resistive term that must be added to the Joule resistance, the difference comparing to the Joule resistance is the existence of partial mutual radiation resistance exactly like mutual inductance.

The capacitance of the model can be calculated by (19), (20) and (22), but the potential coefficients include the retardation term:

$$P_{kk'} = \frac{1}{4\pi\varepsilon S_k S_{k'}} \int_{C_k} \int_{C_k} \frac{e^{-J\rho |x-x'|} ds_k ds_{k'}}{|x-x'|}$$
(36)

Similarly to the inductive model, a resistance must be putted in series with the capacitance. Then, the complete model including retardation is represented in figure 3.



Figure 3. rPEEC equivalent circuit

Another model including retardation has been proposed by A. Ruehli. It consists in replacing partial mutual inductances and capacitance by voltage sources [1].



Figure 4. rPEEC model proposed by A. Ruehli.

Where:

$$u_i^{(L)}(t) = \sum_{j \neq i} L p_{ij} \frac{\partial i_j(t - t_{ij})}{\partial t}$$
(37)

$$u_{i}^{(c)}(t) = \sum_{j \neq i} \frac{p_{ij}}{p_{jj}} v_{j}(t - t_{ij})$$
(38)

 $Z_{in} = 73.1 + j42.5 \ \Omega \ .[20]$ 

$$C_i' = \frac{1}{p_{ii}} \tag{39}$$

In order to simulate these retarded voltage sources a history mechanism must be implemented in the circuit simulator.

The current distribution can be found by resolving a matrix equation similar to (29) with complex elements [21]:  $Z_{kk'} = \delta(k,k') \cdot R + j \omega L'_{kk'}$ 

$$+\frac{1}{j\omega} \left( P'_{k+1,k'+1} - P'_{k,k'+1} - P'_{k+1,k'} + P'_{k,k'} \right)$$
(40)

With:

$$L'_{kk'} = Lp_{kk'} e^{-j\beta |x_k - x_{k'}|}$$
(41)

$$P'_{kk'} = P_{kk'} e^{-j\beta |x_k - x_{k'}|}$$
(42)

For power electronics applications, the model of figure 3 is most useful, it can be combined easily with other circuit models (like diodes, transistors...) into input circuit for a circuit simulator like SPICE.

# V. APPLICATIONS

In this section, we present a variety of examples from different application areas. When the wavelength of highest frequency of interest is much longer than the physical dimensions of longest coupling of interest, simple PEEC models for interconnect modeling can be used. However, for microwave applications like antennas, the complete rPEEC model must be applied.

In this section, we will try to study the limits of the quasistatic approach. That's why, we will study the case of the antenna and the case of PCB with a ground plane.

The first example concerns the case of antennas with different values of  $x = \frac{L}{\lambda}$ , the source voltage is placed at the middle of the antenna (figure 5). The current distribution is calculated by the two approaches (PEEC and rPEEC) for x=0.2 and x=0.5, results are represented in figure 6 and figure 7.





For low values of x, the retarded and the non-retarded approaches give the same result (figure 6). However, for important values of x, the results given by the two approaches differ significantly (figure 7).

In order to show that the retarded waveform is correct, we calculate the input impedance by  $Z_{in} = \frac{e_g}{I(0)}$ , we find the known result for a half wave antenna:





In the second example, we treat the case of a microstrip line with a ground plane (figure 8-a) with a load  $Z_L = 50 \Omega$  (figure 8-b).



Figure 8. Microstrip line

In our case, we have  $L = 20 \ cm$ , w = 1mm,  $h = 1.5 \ mm$ .

In order to confirm the rPEEC approach, we compare the input impedance calculated to the impedance deduced from the measurement of reflection coefficient  $s_{11}$  by the relation:

$$Z_{in} = Z_c \frac{1 + s_{11}}{1 - s_{11}} \tag{43}$$

Where  $Z_c$  is given by:

$$Z_c = \frac{60}{\sqrt{\varepsilon_e}} \ln\left(\frac{8h}{w} + \frac{0.25w}{h}\right)$$
(44)

The effective permittivity is given by:

$$\varepsilon_{e} = \frac{\varepsilon_{r} + 1}{2} + \frac{\varepsilon_{r} - 1}{2} \left[ \left( 1 + \frac{12h}{w} \right)^{-0.5} + 0.04 \left( 1 - \frac{h}{w} \right)^{2} \right]$$
(45)

 $\mathcal{E}_r$  is the relative permittivity. In our case:  $\mathcal{E}_r = 4.7$ ,

 $\mathcal{E}_e = 3.28$  and  $Z_c = 82.75 \ \Omega$ . [15]

In figure 9-a and 9-b, calculated impedance is compared to measured impedance.



Figure 9-a. Real part of the input impedance calculation/measurement



The two examples show the validity of rPEEC approach for a large frequency range. In the next section, we will try to find the limits of the non-retarded approach for calculating radiated field.

# VI. LIMITS OF NON RETARDED APPROACH

In order to find the limits of the PEEC approach, we consider a rectangular loop 10x10cm. As a first step, we study the input impedance of the loop, simple model including just inductive phenomenon is compared to PEEC and rPEEC models.

For the simple inductive model, we consider the loop as an inductance  $L_b = 250 \ nH$ .

For both PEEC and rPEEC approaches, current distribution must be calculated (figure 10-a, 10-b), after that

the input impedance is deduced by 
$$Z_{in} = \frac{e_g}{I(0)}$$

For low values of  $L/\lambda$ , we see that the current is almost constant along the loop. Then, the inductive model suffices. For more important values of  $L/\lambda$ , the current distribution can't be considered as constant, otherwise the difference between PEEC and rPEEC approaches for current distribution isn't very important.



Figure 10-a. current distribution for  $L_1=L_2=\lambda/64$ 



Figure 10-b. current distribution for  $L_1=L_2=\lambda/8$ 

The same remarks can be done for the imaginary part of input impedance (figure 11).







Figure 12. Difference between inductive model and complete model

Owing to figure 12, we see that the difference between the simple inductive model and the complete model is more than 10% from a perimeter of  $\lambda/7$ .

In order to see the importance of the retardation effect, we calculate the magnetic field radiated by the loop at a distance z=10cm (figure 13).



Figure 13. magnetic field radiated by a loop.

Field is calculated by three approaches:

 $1^{st}$  approach: We consider the current as constant on the loop:  $I = \frac{e_g}{L_b \omega}$ , by neglecting the retardation effect, we

obtain the vector potential by: .. .11

$$\vec{A} = \frac{\mu}{4\pi} I \sum_{i} \frac{dl_i}{r_i}$$
(46)

 $2^{nd}$  approach: The current is constant but retardation effect is not neglected, we obtain the vector potential by:

$$\vec{A} = \frac{\mu}{4\pi} I \sum_{i} \frac{e^{-j\beta r_i} dl_i}{r_i}$$
(47)

<u>3<sup>rd</sup> approach</u>: The current is calculated by the rPEEC approach and the vector potential by:

$$\vec{A} = \frac{\mu}{4\pi} \sum_{i} I_i \frac{e^{-j\beta r_i} dl_i}{r_i}$$
(48)

In figure 14, we compare fields calculated by the three approaches for the regarded loop.



Figure 14. effect of retardation on magnetic field.

We can see that since a frequency  $f=1.5 \times 10^8 Hz$ , neglecting retardation and propagation effects involve an error more than 15% on the magnetic field. This frequency corresponds to a ratio:  $x = \frac{L}{\lambda} = \frac{L \cdot f}{c} = \frac{0.4 \times 1.5 \times 10^8}{3 \times 10^8} = 0.2$ , where L is the perimeter of the loop (in our case  $L = 4 \times 0.1 = 0.4m$ ).

## VII. CONCLUSION

The principle of the partial element equivalent circuit (PEEC) for interconnect modeling is reminded. Different approaches for including retardation effect are presented. The validity of the rPEEC approach is proved for two different examples which are antenna and microstrip line. Limits of the quasi-static approach are studied for calculating input impedance and radiated magnetic field.

We can conclude that effect of retardation and propagation must be taken into account from a circuit dimension L= $\lambda/5$ .

#### REFERENCES

- [1] A. E. Ruehli, H. Heeb "Three dimensional interconnect analysis using partial element equivalent circuit" IEEE Transaction on Circuits and Systems, vol.39, nº11, November 1992.
- A. E. Ruehli, "Equivalent circuit models for dimensional [2] multiconductor systems" IEEE Transaction on Microwave Theory and Techniques, vol. 22, Mars 1974.
- A. E. Ruehli, P.A. Brennan "efficient capacitance calculations for [3] three dimensional multiconductors systems" IEEE Transaction on Microwave Theory and Techniques, vol.21,  $N^0$  2, February 1973.
- [4] A. E. Ruehli, P.A. Brennan "Capacitance models for integrated circuit metallization wires" IEEE Journal of solid-state circuits, vol.SC-10, N<sup>0</sup> 6, December 1975.
- [5] A. E. Ruehli, "Survey of computer aided electrical analysis of integrated circuit interconnections" IBM Journal on R&D, vol.23, N<sup>0</sup>6. November 1979.
- [6] P. D. Patel, "Calculation of capacitance coefficients for a system of irregular finite conductors on a dielectric sheet" IEEE Transaction on Microwave Theory and Techniques, vol.MTT.19, Nº11, Nov 1971.
- [7] P. Silverter, P. Benedek, "Equivalent capacitances of microstrip open circuits" IEEE Transaction on Microwave Theory and Techniques, vol.MTT.20, Nº8, August 1992.
- [8] A.E. Ruehli, G. Antonini, J. Esch, J. Ekman, A. Mayo, A. Orlandi, "Nonorthogonal PEEC Formulation for Time and Frequency-Domain EM and Circuit Modeling" IEEE Transaction on Electromagnetic Compatibility, VOL. 45, NO. 2, MAY 2003.
- G. Wollenberg, A. Gorisch, "Analysis of 3-D Interconnect Structures [9] with PEEC Using SPICE" IEEE Transaction on Electromagnetic Compatibility, VOL. 41, NO. 4, NOVEMBER 1999.
- [10] K. M. Coperich, A. E. Ruehli, A. Cangellaris "Enhanced Skin Effect for Partial-Element Equivalent-Circuit Models" IEEE Transaction on Microwave Theory and Techniques, vol.48, Nº9, September 2000.

- [11] P. J. Restle, A. E. Ruehli, S. G. Walker, G. Papadopoulos "Full-Wave PEEC Time-Domain Method for the Modeling of On-Chip Interconnects" IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, VOL. 20, NO. 7, JULY 2001.
- [12] P. F. Lopez, C. Arcambal, Y. V. Gilabert, A. Ramanujan, D. Baudry, A. Louis, B. Mazari "Development of magnetic near-field model and insertion into a commercial electromagnetic simulator" Turkish Journal of Electrical Engineering & Computer Sciences, VOL. 17, NO. 3, pp289-300, November 2009.
- [13] M. Youssef, J. Roudet, Y. Marechal "Near-Field characterization of Power Electronics circuit for radiation prediction" 28<sup>th</sup> IEEE Power Electronics Specialists Conference PESC'97, June 1997.
- [14] M. Youssef, G. Antonini, E. Clavel, J. Roudet, S. Cristina, A. Orlandi "Conducted and radiated EMI characterization on Power Electronics Converters" IEEE International Symposium on Industrial Electronics ISIE'97, Guimaraes, Portugal, July 1997.
- [15] J. Roudet, J.L. Schanen, P. Saguet "Microwave techniques validation in power Electronics Interconnecting Modeling" EMC'96, vol.1, pp144-148, Roma, September 1996.
- [16] S. Daijavad, J. Janak, H. Heeb, A. Ruehli, D. McBride "A Fast Method for computing Radiation from printed circuit Boards" IEEE International Symposium on EMC, pp300-303, 1990.
- [17] C.R. Paul "Introduction to Electromagnetic Compatibility" Whiley and sons, 1993.
- [18] C.R. Paul "Introduction to Electromagnetic Fields" McGraw-Hill, 1987.
- [19] J.D. Kraus "Antennas" McGraw-Hill, 1988.
- [20] W.L. Stutzman, G.A. Thiele "Antenna theory and design" Whiley and sons, 1998.
- [21] R.F. Harrington "Time-Harmonic Electromagnetic Fields" New York McGraw-Hill, 1961.